NAG Toolbox for MATLAB

f08as

1 Purpose

f08as computes the QR factorization of a complex m by n matrix.

2 Syntax

```
[a, tau, info] = f08as(a, 'm', m, 'n', n)
```

3 Description

f08as forms the QR factorization of an arbitrary rectangular complex m by n matrix. No pivoting is performed.

If $m \geq n$, the factorization is given by:

$$A = Q\begin{pmatrix} R \\ 0 \end{pmatrix},$$

where R is an n by n upper triangular matrix (with real diagonal elements) and Q is an m by m unitary matrix. It is sometimes more convenient to write the factorization as

$$A = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix}\begin{pmatrix} R \\ 0 \end{pmatrix},$$

which reduces to

$$A = Q_1 R,$$

where Q_1 consists of the first n columns of Q, and Q_2 the remaining $m - n$ columns.

If $m < n$, R is trapezoidal, and the factorization can be written

$$A = Q\begin{pmatrix} R_1 & R_2 \end{pmatrix},$$

where R_1 is upper triangular and R_2 is rectangular.

The matrix Q is not formed explicitly but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

Note also that for any $k < n$, the information returned in the first k columns of the array **a** represents a QR factorization of the first k columns of the original matrix A.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \geq n$, the elements below the diagonal are overwritten by details of the unitary matrix Q and the upper triangle contains the corresponding elements of the n by n upper triangular matrix R.

If $m < n$, the strictly lower triangular part contains details of the unitary matrix Q and the remaining elements are overwritten by the corresponding elements of the m by n upper trapezoidal matrix R.

The diagonal elements of R are real.

2: **tau($*$) – complex array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

further details of the unitary matrix Q.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}n^2(3m - n)$ if $m \geq n$ or $\frac{8}{3}m^2(3n - m)$ if $m < n$.

To form the unitary matrix Q f08as may be followed by a call to f08at:

```
[a, info] = f08at(a, tau);
```

but note that the second dimension of the array **a** must be at least **m**, which may be larger than was required by f08as.

When $m \geq n$, it is often only the first n columns of Q that are required, and they may be formed by the call:

```
[a, info] = f08at(a(:,1:n), tau);
```

To apply Q to an arbitrary complex rectangular matrix C, f08as may be followed by a call to f08au. For example,

```
[c, info] = f08au('Left','Conjugate Transpose', a, tau, c);
```

forms $C = Q^{\mathrm{H}}C$, where C is m by p.

To compute a QR factorization with column pivoting, use f08bs.

The real analogue of this function is f08ae.

9 Example

```
a = [complex(0.96, -0.8100000000000001), complex(-0.03, +0.96), complex(-
0.91, +2.06), complex(-0.05, +0.41);
      complex(-0.98, +1.98), complex(-1.2, +0.19), complex(-0.66, +0.42),
...
     complex(-0.8100000000000001, +0.5600000000000001);
       complex(0.62, -0.46), complex(1.01, +0.02), complex(0.63, -0.17),
complex(-1.11, +0.6);
      complex(-0.37, +0.38), complex(0.19, -0.54), complex(-0.98, -0.36),
complex(0.22, -0.2);
       complex(0.83, +0.51), complex(0.2, +0.01), complex(-0.17, -0.46),
complex(1.47, +1.59);
              complex(1.08,   -0.28),   complex(0.2,   -0.12),   complex(-
0.07000000000000001, +1.23), complex(0.26, +0.26)];
[aOut, tau, info] = f08as(a)
```

```
aOut =
   -3.0870               -0.4885 - 1.1417i    0.3774 - 1.2437i   -0.8552 -
0.7073i
   -0.3270 + 0.4238i    1.5163                1.3731 - 0.8176i   -0.2509 +
0.8203i
    0.1692 - 0.0798i   -0.4537 - 0.0065i   -2.1713               -0.2273 -
0.2957i
  -0.1060 + 0.0727i   -0.2734 + 0.0978i   -0.2918 + 0.4888i   -2.3534
    0.1729 + 0.1606i   -0.3236 + 0.1230i    0.2728 + 0.0470i    0.7054 +
0.2515i
    0.2699 - 0.0152i   -0.1646 + 0.3389i    0.5348 + 0.3988i    0.2703 -
0.0727i
tau =
   1.3110 - 0.2624i
```

```
   1.1051 - 0.4504i
   1.0403 + 0.2122i
   1.1860 + 0.2012i
info =
           0
```